

GEOMETRY PLAYGROUND

Activities | Grades 6–8

www.exploratorium.edu/geometryplayground/activities

EXPLORING TESSELLATIONS

Background: What is a tessellation?

A *tessellation* is any pattern made of repeating shapes that covers a surface completely without overlapping or leaving any gaps. A checkerboard is a tessellation made of squares. The squares meet edge to edge with no gaps and no overlapping areas. The pattern of bricks on a wall is a tessellation made of rectangles.

Over 2,200 years ago, ancient Greeks were decorating their homes with tessellations, making elaborate mosaics from tiny, square tiles. Early Persian and Islamic artists also created spectacular tessellating designs. More recently, the Dutch artist M. C. Escher used tessellation to create enchanting patterns of interlocking creatures, such as birds and fish.

Making tessellations combines the creativity of an art project with the challenge of solving a puzzle.

Part One: Making a Translation Tessellation

[45 minutes]

Suppose you wanted to cover a floor with tiles. You could cover it with square tiles, since squares fit together without leaving any gaps.

In this activity, you're going to transform a rectangle into a more interesting shape, then make a tessellation by repeating that shape over and over again.

Materials:

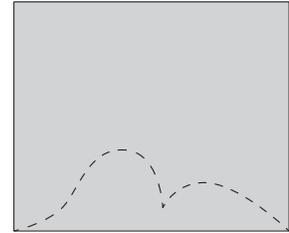
- Index card 3" x 5"
- Ruler
- Scissors
- Blank paper
- Pencil
- Transparent tape
- Colored markers or pens
- 2.5" x 3" Grid paper (included)
- Optional: Printed example of a tessellation to show students before starting activity. (Possibly from the online M. C. Escher gallery at: <http://www.mcescher.com/Gallery/gallery-recogn.htm>)

Try This:

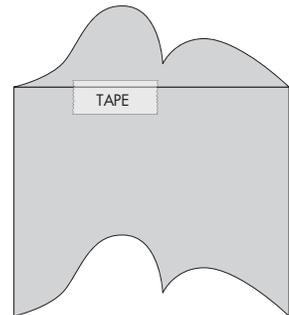
Step 1 Cut an index card in half, creating a 2.5" x 3" rectangle.

Step 2 Find the area of the rectangle (length x width).

Step 3 Draw a line between two adjacent corners on one of the long sides of the rectangle. Your line can be squiggly or made up of straight segments. Whatever its shape, your line must connect two corners that share one side of the rectangle.

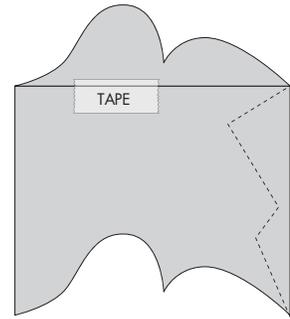


Step 4 Cut along the line you drew. Take the piece you cut off and slide it straight across to the opposite long side of the rectangle. Line up the long, straight edges of the two pieces and tape them together.

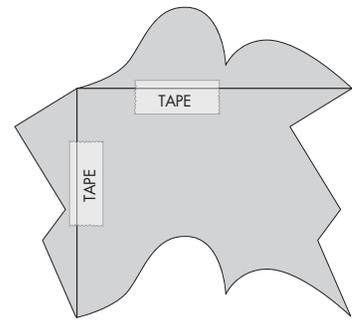


Step 5 Can you tessellate with this shape? Try tracing this shape several times, creating a row going across a piece of paper. Line up the cut edges of the shape as you trace it.

Step 6 Now draw another line that connects two adjacent corners on one of the short sides of the shape.



Step 7 Cut along this new line. Take the piece you cut off and slide it straight across to the opposite side of the shape. Line up the straight edges and tape them together.



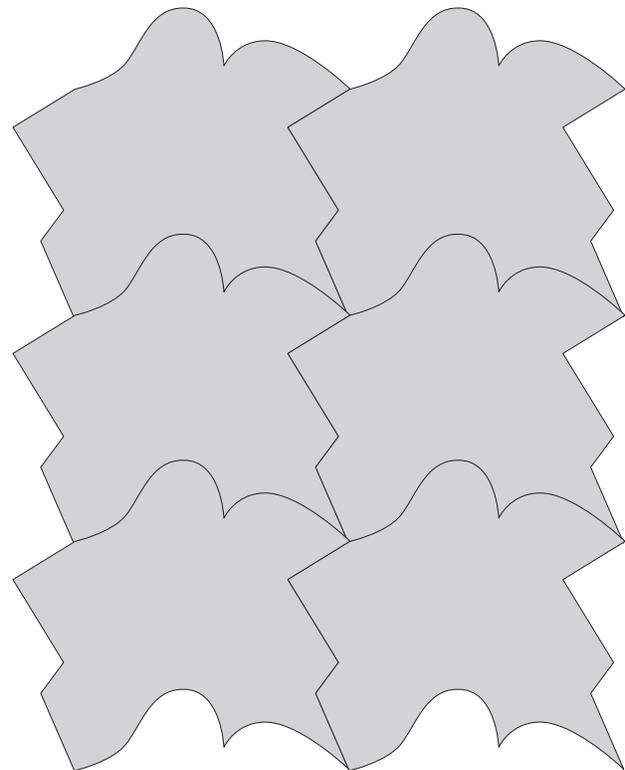
Step 8 You have now created a shape that you can use as a pattern to make a tessellation. What's the area of this shape? Write the letter A on one side of the shape and turn it over and write the letter B on the other side.

Step 9 On your grid paper, carefully trace around your pattern shape.

Can you figure out where to place the pattern so that your paper will be covered with repetitions of this shape with no overlaps and no gaps? Try to cover your whole sheet of paper by tracing the pattern, moving it, then tracing it again.

If you start with side A facing up do you ever have to turn it over to side B to make your tessellation? If you only have to slide the piece without flipping it over or rotating it, then you are making a *translation* tessellation.

In math, translation means shifting the position of a shape without moving it in any other way.



Step 10 Look for a clever way to color in the resulting design on your sheet of paper.

Does your shape look like a fish? A bird? An elephant?

Part Two: Making a Different Kind of Tessellation

[30 minutes]

You've made translation tessellations by tracing a pattern, then sliding it to a new position and tracing it again. Now you can make a different kind of tessellation by turning (*rotating*), or by flipping over (*reflecting*) the pattern to a new position before copying it again.

Materials:

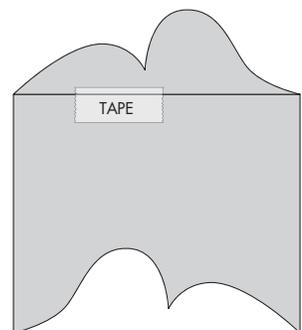
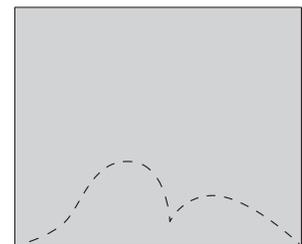
- Index card, 3" x 5"
- Ruler
- Scissors
- Blank paper
- Pencil
- Transparent tape
- Colored markers or pens
- 2.5" x 3" Grid paper (included)

Try This:

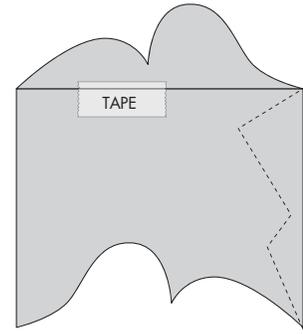
Step 1 Cut an index card in half, creating a 2.5" x 3" rectangle.

Step 2 Draw a line between two adjacent corners on one of the long sides of the rectangle. Your line can be squiggly or made up of straight segments. Whatever its shape, your line must connect two corners that share one of the long sides of the rectangle.

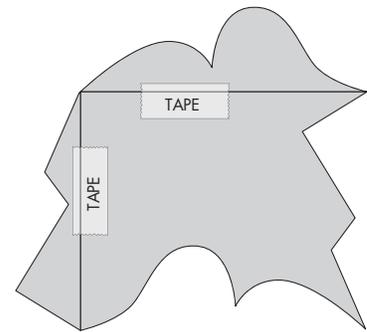
Step 3 Cut along the line you drew. Take the piece you cut off, **flip it over** and then slide it across to the opposite long side of the rectangle. Line up the straight edge of the piece with the straight edge of the opposite edge of the rectangle. Tape the piece in place.



Step 4 Now draw another line that connects two adjacent corners on one of the short sides of the shape.



Step 5 Cut along this new line. Take the piece you cut off, flip it over and then slide it straight across to the opposite side of the shape. Line up the straight edge of the piece with the straight edge of the shape. Tape the piece in place.

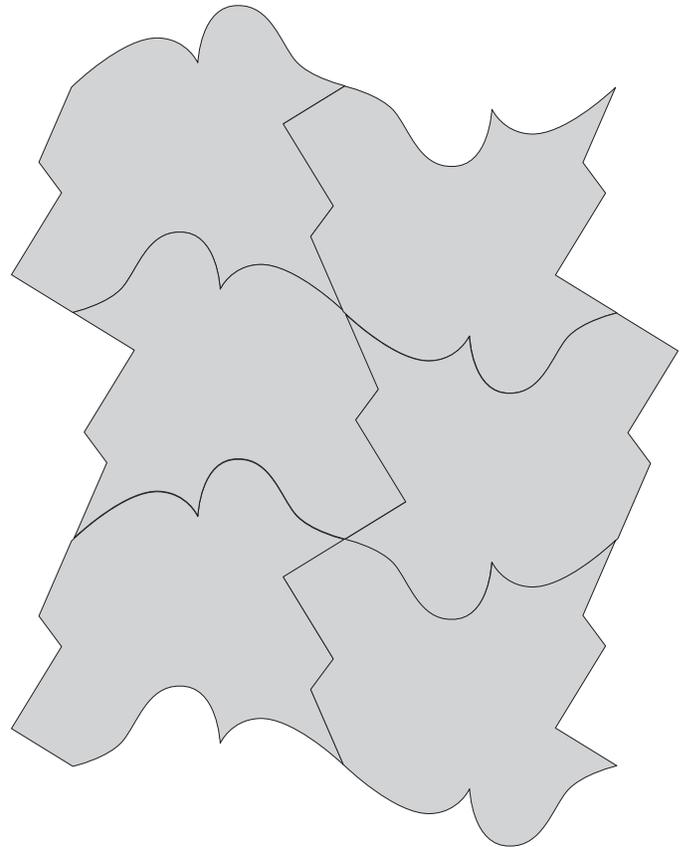


Step 6 You have created a shape that you can now use as a pattern to make a tessellation. Write the letter A on one side of the pattern, then turn it over and write the letter B on the other side.

Step 7 On your grid paper, carefully trace around your pattern shape. It may help to position the squared-off corner (formerly the edge of the index card) in one corner of the grid.

Can you figure out where to place the pattern piece so that your paper will be covered with repetitions of this shape with no overlapping and with no gaps? Try to cover your whole sheet of paper by tracing the pattern, moving it, then tracing it again. If you start with side A facing up, do you ever have to turn it over to side B to make your tessellation? If you have to flip your piece over, you are making a reflection tessellation. If you also had to move the piece to a new position, you have used translation.

Step 8 Look for a clever way to color in the resulting design on your sheet of paper.



When you cut a shape out of paper, then flip it over, the flipped shape looks like a mirror image of the original shape. So a tessellation made with this technique is called a *reflection* tessellation.

Your hands can help you understand the concept of mirror reflection. Your two hands are the same shape—but your right hand is a mirror reflection of your left hand (and vice versa.)

Part Three: Making a Rotation Tessellation

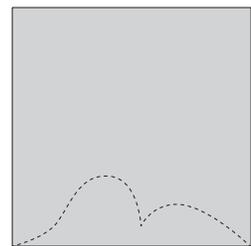
[30 minutes]

Materials:

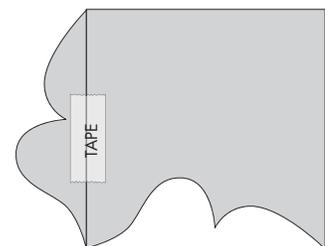
- Index card
- Ruler
- Scissors
- Blank paper
- Pencil
- Transparent tape
- Colored markers or pens
- 2.5" Square Grid paper (included)

Try This:

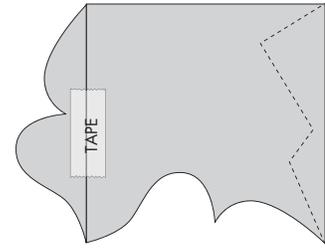
- Step 1 Draw a 2.5" x 2.5" square on your index card.
- Step 2 Cut out the square from the index card.
- Step 3 Draw a line between two adjacent corners on one side of the square. Your line can be squiggly or made up of straight segments. Whatever its shape, your line must connect two corners that share one side of the square.



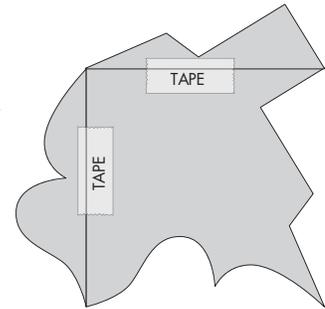
- Step 4 Cut along the line you drew. Take the piece you cut off (without flipping) and slide it to an adjacent side of the square. Line up the straight edges and tape them together.



Step 5 Now draw another line that connects the two corners on the side adjacent to the cut side of the square.

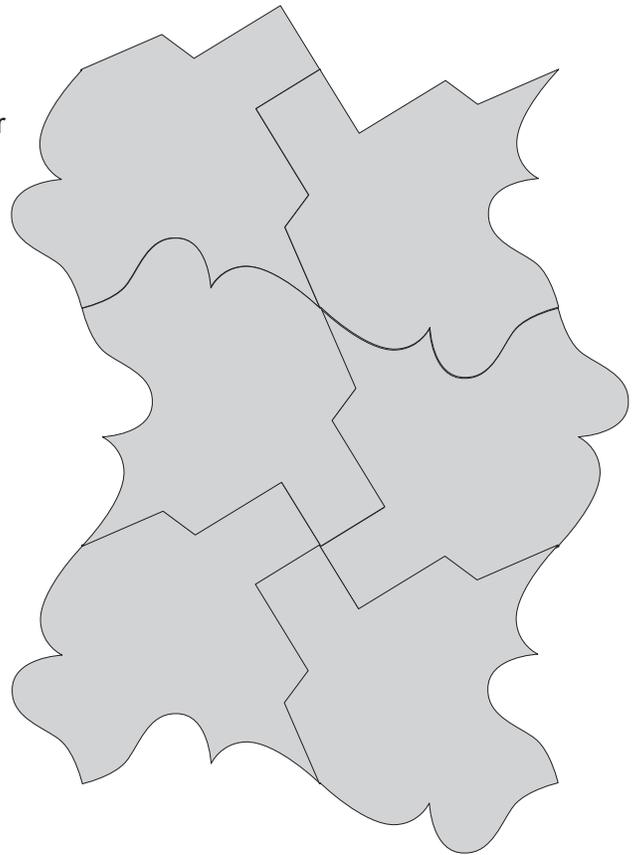


Step 6 Cut along this new line. Take the piece you cut off (without flipping) and slide it to its adjacent side. Line up the straight edge of the cut piece with the straight edge of the square, and tape them together.



Step 7 You have now created a shape you can use as a pattern to make a tessellation. Write the letter A on one side of the shape and turn it over and write the letter B on the other side.

Step 8 On your grid paper, carefully trace around your pattern piece. Try to cover your whole sheet of paper by tracing the pattern, then moving it and tracing it again. If you start with the side A facing up, do you ever have to turn it over to side B to make your tessellation? If you have to flip your piece over, you have made a reflection tessellation. If you also had to move the piece to a new position you have also used translation. If you have to turn or rotate the shape to make your tessellation, then you have made a rotation tessellation.



Step 9 Look for a clever way to color in the resulting design on your sheet of paper.

What's Going On?

If you look around, you'll see many repeating patterns: on wallpaper, on fabric, in a tiled bathroom floor. In this activity, we explored some of the mathematical rules used to make repeating patterns. Tessellations are patterns that cover a surface completely without overlapping or leaving any gaps.

The three mathematical rules of repetition used in this activity are:

- Shifting the position of a shape (something mathematicians call translation.)
- Rotating a shape to a new position (mathematicians don't have a fancy name for this; they just call it rotation.)
- Flipping a shape over so it looks like a mirror reflection of itself (mathematicians call this reflection.)

Many people think of M. C. Escher's work when they think of tessellations. (<http://www.mcescher.com/Gallery/gallery-recogn.htm>) Escher worked for a long time on each of his designs before he was satisfied with the final result. After you try this activity once, you may want to make several more pattern pieces until you come up with an especially interesting or evocative design.

People don't usually think of math when they look at beautiful designs, but math provides tools that help us create and analyze patterns. The three rules used to create these tessellations—translation, rotation, and reflection—are used in making many of the repeating designs we see around us every day.

Going Further:

To view the work of M. C. Escher, go to:

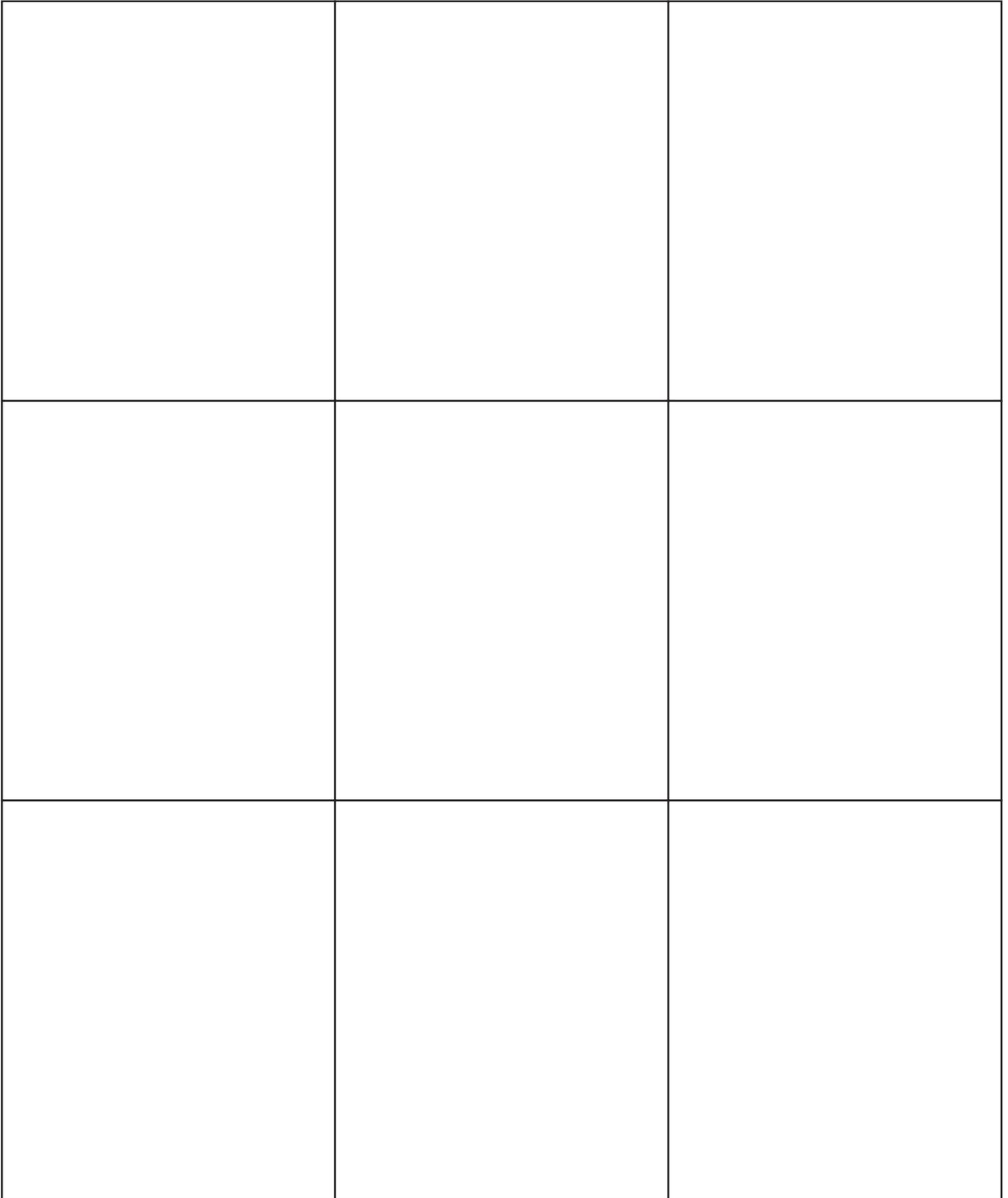
<http://www.mcescher.com/Gallery/gallery-recogn.htm>

To see tessellations that other students have made, go to:

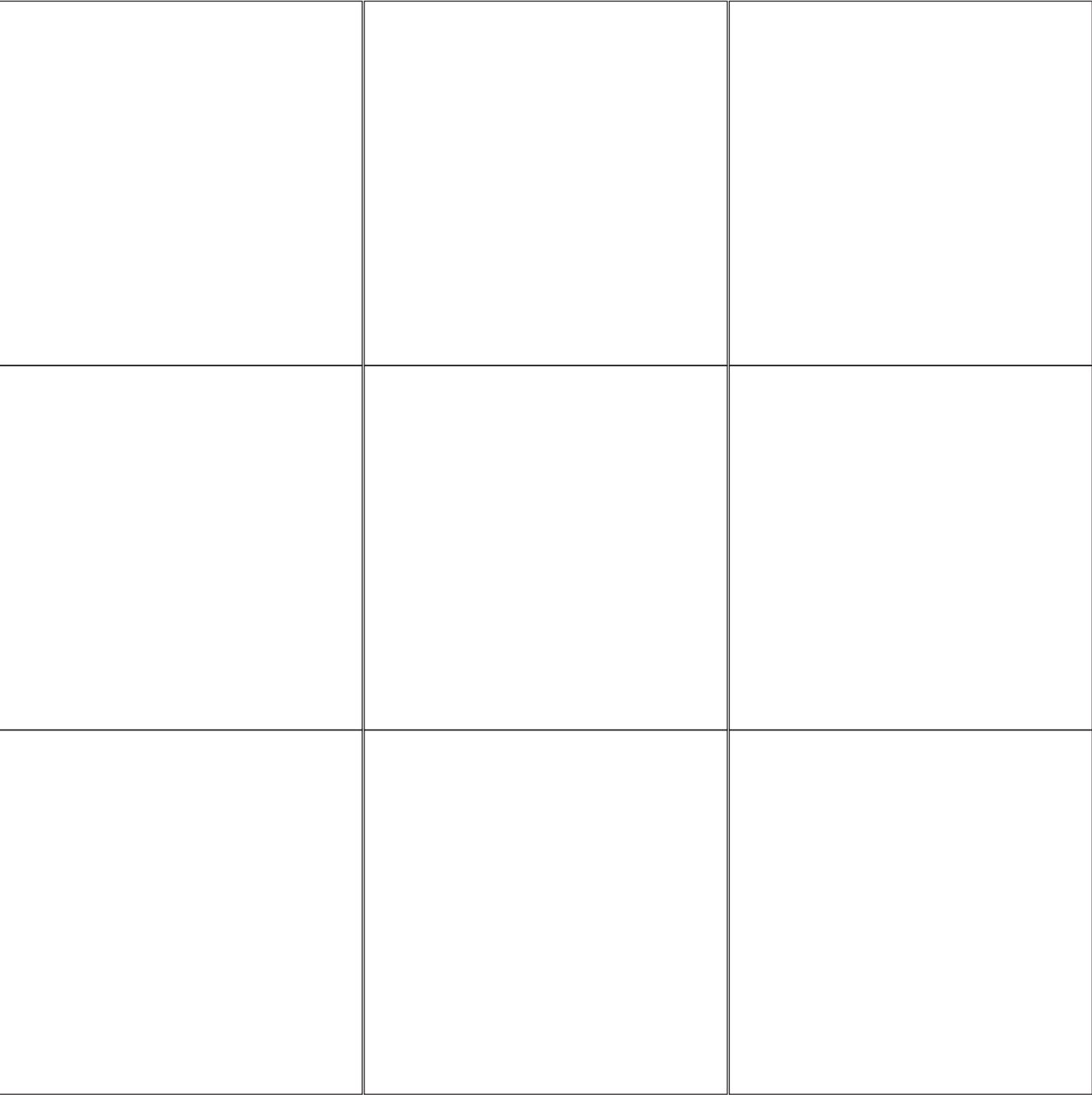
<http://www.worldofescher.com/contest/>

The site includes an ongoing contest for the best tessellation, and has a "Hall of Fame" that includes a number of tessellations by middle-schoolers.

2.5" x 3" Grid



2.5" Square Grid



EXPLORING TESSELLATIONS

Apply transformations and use symmetry to analyze mathematical situations:

- Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.

Use visualization, spatial reasoning, and geometric modeling to solve problems:

- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Apply appropriate techniques, tools, and formulas to determine measurements:

- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision.