MTC Penn Summer Immersion Workshop
June 12 -14, 2017
https://mtcpenn.wixsite.com/mathcircle

Lockers: An open and shut case

Many of you are familiar with the locker problem:

Problem 1. At a large high school, there are 10000 lockers all on one wall of a long corridor. The lockers are numbered, in order, 1, 2, 3, … , 10000, and to start, each locker is closed. There are also 10000 students, also numbered 1, 2, 3, … , 10000. The students walk the length of the corridor, opening and closing lockers according to the following rules.

1. Student 1 opens every locker.
2. Student 2 closes every second locker.
3. Student 3 changes the state of every third locker, closing it if it is open, and opening it if it is closed.

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k. Student k changes the state of every k-th locker

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a. When all 10000 students have walked the corridor, which lockers end up open?

b. If the students go down in a different order, is the result changed?

c. What if student 3 is ill and had to skip her turn? What if she took a second turn when the teacher was not looking?

d. What if students 3 and 9 are ill? 3 and 10?

Problem 2. Suppose that we can send any students we like down the corridor. If, when we are done, we want only locker 1 open and all others closed, then which students should go?

Problem 3. Suppose that we want only the lockers with prime numbers open. Which students should be sent down the corridor?

Problem 4. A number is called square-free if it is not divisible by the square of any prime number (so the number 1 is square free.)

a. List the first 15 square-free numbers.

b. If we send all of the students with square free numbers down the corridor, which lockers will be open when the activity is done?

Problem 5. Suppose we want only locker 3 open. Which students should be sent down the corridor? What if we want only locker 9 open? Both lockers 3 and 9 and no other lockers?

Problem 6. Suppose that we send down the corridor exactly those students with perfect square numbers (e.g., 1, 4, 9, 16,….) Which lockers are left open when this activity is concluded? What if we send only the students with numbers that are perfect cubes?

Problem 7. Suppose that we send only those students with perfect nth power numbers (e.g., 1, 2!, 3!, …) Which lockers are left open when this activity is concluded?

Problem 8. If when we are done we want only cube lockers to remain open, which students should we send? Fourth powers open? Fifth?

Problem 9. Whom shall we send so that only prime numbered lockers remain open?